LEONARDO
&
THE RECIPROCAL STRUCTURES
SPATIAL RECIPROCAL STRUCTURES

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SUMMARY

The Spatial Reciprocal Frame (S.R.F.) is a three-dimensional grid, mainly used as a roof structure, consisting of a system of mutually supporting beams. The beams, being of relatively short dimension, hold each other up through simple contact bindings and are capable of creating various types of construction. This building concept allows the dismantling of the framework and its subsequent reassembly into different forms. Moreover, these geometrical and functional flexibilities consent the S.R.F. to meet ecological building standards of environmental regulations. The irregular arrangement of their modular support points and their ability to be dismantled make them suitable even in unusual situations; for example, to provide the covering of an archaeological field with a temporary roof.

This paper summarises the geometrical research carried out at the University of Bergamo on the S.R.F. and describes the setting-up of a physical Finished Elements Model (FEM) created to test the structural potential of the system. Finally, the project submitted to the Italian Pavilion competition for the 2010 Shanghai World Expo is presented and demonstrates the practical and viable use of S.R.F. in a contemporary architectural context.

1. INTRODUCTION

Spatial Reciprocal Frames (S.R.F) are three-dimensional structures created with beams that hold each other up through simple support bindings. The construction principle is antique in origin and citations to its use have been found in the manuscripts of medieval treatise writers, such as Villard de Honnecourt, and in the documents of renaissance architects including those of Sebastiano Serlio and Leonardo da Vinci's Atlantic Code.

Their use today appears to present particular interest for the planning of temporary buildings that can be dismantled and their materials recuperated (demountable buildings). What is more, they hold specific implications for eco-sustainable building systems; in particular, for their potential reuse in the creation of both large and small structures that can be repeatedly readapted for changing uses. In such cases, the resulting constructions may be of distinct forms and dimensions, there need not be waste of the previously used materials, or the consumption of additional ones.

Indeed, a key characteristic of these structures is that they are able to reform junctions with different geometries that can then be dismantled and reformed again using differently positioned support points that are not necessarily coplanar. The same elements (i.e. the beams) can then be used to create junctions formed of three, four, five or even more connecting beams with a static system equilibrium that changes depending upon the variation of the stipulated heights of the base support points. This geometrical and structural flexibility also renders the S.R.F. subject to particular interest with regards to the LEED (Leadership in Energy and Environmental Design) certification, in which protocols are assigned a higher value of eco-sustainability when the structure can be reused at the end of its functional life in such a way that it can be reassembled into alternative forms and compositions for purposes that differ to its initial use.

The structure lends itself as being particularly well suited for the covering of large spaces and can be built using highly diverse materials, including, steel, lamellar wood and even reinforced concrete that can be readily employed. Such flexibility makes it appealing when special solutions are required, as is the case when creating of temporary and reversible protections for archaeological excavations sites, where the ability to dismantle the system is of particular importance, as is the ability to stipulate the position of the base support points with total geometrical flexibility, both in the horizontal and in the vertical plane, in a way that the structure does not interfere with the historical remains.

The cases reported here are essentially the studies and laboratory prototypes that have been perfected through the course of the research conducted within the Faculty of Engineering at the University of Bergamo between 2006 and today; the results of which have permitted the elaboration of a project that has been submitted to the competition for the
construction of the Italian Pavilion at the Shanghai 2010 World Expo.

Figure 1. The study of a standardised element for structural junctions composed of 3, 4 and 6 elements.

2. HISTORICAL REFERENCES

The construction of roofs and ceilings using "short" beams is recognised as being one of the most antique technologies, so much so that the origin of their use has become somewhat lost within the depths of construction traditions.

Founded upon the vast experience of man's use of natural materials, like wood, it was inevitable that load-bearing elements were initially limited to the dimensions found in nature. This antique method continues to form part of the material know-how of some "primitive" populations today and its use can be found within the mobile dwellings of tribesmen in Lapland and in North America.

But, history has also recorded the presence of similar construction procedures in classical architectural productions. On the site of the ancient fortified city of Nisa, Mesopotamia, within present day Afghanistan, Russian archaeologists have identified the use of such structures that date as far back as the second millennium BC; Pizzetti found evidence indicating that the ceiling of the "Square Hall" was created according to a scheme of "short beams". Interesting evidence has also been found in the stone temples excavated into the rock at Bany Jahn, where structures typical of the ceilings of the wooden temples (that are believed to have been built with systems of short beams that auto-supported one another) are reproduced on the intrados of the vaulted ceilings.

Moreover, a description made by Julius Caesar about the bridge he built over the Rhine refers to the use of short beams for its construction; using this account, hypothesised plans were produced by Massimo Scolari for its reconstruction and presented in a recent exhibition in Vicenza.

Villard de Honnecourt, the medieval engineer and constructor of cathedrals, describes the method in his "notebook" (1225-1250), presenting it as the method to "construct a house or a tower when the wooden beams are too short".

In the same historical period, the use of this construction technique was used for building the temples of the Chogen Buddhist monks (1121-1206), and the so-called "Rainbow Bridge", in Shandong, China, where, in the paintings that have often represented it as being one of the most important architectural works created during the Song dynasty (960-1280), one clearly notes that the series of beams appear to follow a scheme of reciprocal support.

Merit must be awarded, however, to Leonardo da Vinci for the most interesting project, described on sheet 899 of the Atlantic Code (ex. sheet 328), for a grid structure "che fa la volta per il tutto", meaning "that makes the vault of them all" (today we would say "geodetic") (see Fig 4), intended as a solution for a temporary pavilion. In numerous other sheets, Leonardo continues projects for bridges and roofs that would be built with such self-supporting beams and indicates their exact geometrical dimensions and construction specifics. In these designs, Leonardo certainly allows us to deduce that the technical and scientific knowledge was already alive in the unwritten traditions of the previous medieval masters and engineers in conjunction with the other antique technologies. It is also clear that the innovation perfected by Brunelleschi for the building of the dome of Florence's Cathedral, La Basilica di Santa Maria del Fiore, took inspiration from these systems and was conceived such that the courses of bricks within the vault were arranged so that they supported each other, thus avoiding the necessity for scaffolding and centring. This Brunelleschian procedure, referred to as "fish backbone" or "herringbone", involves the creation of a series of spiralled schemes within the layering of the bricks that allows the load of a vault to be discharged upon the inferior spirals; thus creating a spatial system that is very similar to a "reciprocal structure". Without wanting to delve deeper into a structural discussion of the Florentine dome, it is nevertheless opportune to underline how the depth of construction knowledge of the spatial reciprocal structure was much greater in the past than what might be presumed if only its applications in "experimental uses" of today were considered; contemporary applications have, on the whole, been for the construction of buildings with exhibition functions or for "exceptional", albeit temporary, houses, like the 1978 houses designed by the Dutchman Bijnen, patented with the name "Nomadome", or the SIGMA system of Gat in 1992. There are also the proposals of the American patent by Crooks in 1980 for constructions made with triangular...
elements “without form” (“a-shaped”), the retractable elements of Baverel and Saidani, and even the wooden dwellings of Graham Brown built in Scotland in the last decade.

**Figure 2 (left).** Sebastiano Serlio; from the first of the five books of architecture.  
**Figure 3 (right).** Leonardo da Vinci; a bridge built using short beams, from the Atlantic Code. (Sheet 286 r-a, part.)

Indeed, the traditional use of the reciprocal structure can be observed throughout technological history, and it was consolidated much more that one might presume if one were only to consider their use in the “experimental” works of today; it is enough to reflect on how their use, other than the numerous examples reported by Leonardo, stands out as being a true and genuine “handbook solution” in the treatise (manual of architecture) of Sebastiano Serlio (1475-1554) when it is necessary to build ceilings and roofs of dimensions larger than those of the available beams.

Even if the study and knowledge of these structures commenced back in the late 1500’s, it seems to have remained in second place as a viable building solution compared to the simple technologies of the wooden trusses (the so-called “Palladian”); not to mention the development of vaulted brick ceilings that followed. The wooden truss method (for various reasons) perhaps represents a more direct evolution in the use of short beams. Nevertheless, if the reciprocal structure were to receive a new surge of interest today, it is certain that it would be due to its flexible and, above all, reversible characteristics (i.e. to the fact they can be readily dismantled and subsequently reused, thus meeting the needs of structures intended as being temporary); not to mention the suggestive and plastic forms that can be created and the architectural image that such structures are able to project.

3. **DEFINITION OF THE SPATIAL RECIPROCAL STRUCTURE**

“Reciprocal” refers to the fact that such structures are composed of various elements (referred to here as “short beams”) that structurally interact through simple support bindings in order to create more complex structures (e.g. roofs and ceilings) of dimensions much greater than the single elements from which they are composed. The abovementioned historical references, so distant in time, offer little that can be transported into the technological environment of today where new materials, such as composites, lamellar woods and steel, appoint these structures greater potentials for their use and development than would have been feasible within the scope of antique traditions. Neither the Puppet Theatre in Seiwa, Japan, built by Kazuhiro Ishii, constructed in wood, nor even the roof of the Louis Kahn room in the Community Centre of Mill Creek, Philadelphia (four self-supporting cavity beams in reinforced concrete) constitute a sufficient enough reference able to emphasize the construction capability of reciprocal beams, that is de-mountable, reversibility and geometrical flexibility. There are, however, the studies of Leonardo, from which we started and that provide the most useful suggestions.

If, in the horizontal plane, two lone beams can theoretically exert reciprocal actions of mutual self-support, in space, the elemental junction of such a structure would require three of more beams to interact according to the composite grids and schemes, which may also vary greatly depending upon the shape and the structural characteristics of the construction. In this way, a junction of three “reciprocal” elements creates an isostatic structure, able to transfer the forces from one beam to the other until the strains are discharged upon the external supports.

It is thus the nature of the bonds between the elements that gives rise to the kind joint that in turn implements the structural continuity of the entire spatial lattice, and the reason why the behaviour of the structure can be considered and analysed as a lattice framework that is plastically coherent and unitary.
4. GEOMETRICAL OPTIMISATION OF THE SHORT BEAMS

Research into spatial reciprocal frames started with the design and construction of the elemental beam and, in particular, with the geometrical optimisation of the beam shape in order to optimise its transversal section and render it resistant to the forces under which the structure would be placed. The prototypal beam considered materials different to those traditionally used for such structures (that would be solid wood); this was due to need of being able to create a beam that was longer than usual in the vertical section, and thus better able to withstand the strains that the structure would be put under, and a shape that would provide improved solutions for bilateral bindings. A concrete of high resistance reinforced fibre was adopted for use in the model and the prototypal study; although, alternative and interesting advantages could also have been obtained using other composites, lamellar stratified woods, or steel profiles. The use of a standardised elemental beam was motivated by the need to guarantee the characteristics of flexibility (regarding the shapes that could be constructed), reversibility and the ability of the constituent elements to be reused and formed into another construction. The shape of the longitudinal section is such that the space above the beams gives rise to a geodetic shape: a spherical cap who’s “indicator” of fullness appears much greater, the greater the horizontal dimension of the structure.

Figure 5. A three-element junction.

The double S form of this element is derived from the necessity to reduce the overlap between the beams without reducing the area of the resistant section, thus avoiding the typical joint of the Serlian beams, that would weaken the structure, and thus overcoming the necessity to excessively incline the beams, as occurred in various contemporary situations (K. Ishii, G. Brown, etc.)

The physical model was built using a high resistance concrete of reinforced fibre and appropriate dimensions and weights (25 Kg), such that it could be easily positioned during the trials and in the laboratory models.

Figure 6. Model of the base element.

Figure 7. Behaviour of the model when submitted to a maximal load.

5. THE THEORETICAL AND PHYSICAL MODELS

The static verification of the construction system of “spatially reciprocal beams” was conducted in a theoretical model using numerical analyses. The results of which were then compared to the results of the fracture trials carried out in the laboratories of the University of Bergamo. This comparison was run on the single elements, i.e. the individual short beams, as well as on the minimal three-element junction. The
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results of the fracture trials confirmed the analytical estimates regarding the capability of the system. They were carried out using tradition static equations as well as elaborated using the computer software ABAQUS, performed on individual finished elements as well as elemental junctions of three and four beams. The substantial homogeneity of the results for the various models in terms of maximal loads and their agreement with the results of the experimental fracture trials, demonstrates the good approximation of the static behaviour generalisation of such structures. Moreover, it demonstrates the possibility of studying, in ideal conditions, even more complex structures constructed with this type of beam.

Figure 8. Fracture trials of a three-element junction in the laboratory of the Faculty of Engineering, University of Bergamo.

Figure 9. The theoretical model (FEM) of a three-element junction.

6. VERIFYING THE RE-ASSEMBLY OF THE PROTOTYPE

A small series of high resistance reinforced fibre concrete beams were produced, identical in form to that analytically studied, and a prototype roof structure was assembled. The use of these elements demonstrated the facility of their assembly and of their dismantling into the different possible grid structures, with junctions of three or four beams and with different geometries, that were orthogonal, triangular and hexagonal. The same elements were then reused and assembled according to a structural scheme of a “Fullerian” dome, i.e. built upon the geometry of a pentagon surrounded by hexagons.

Figure 10. The physical model with beams assembled into hexagonal schemes with three-element junctions.

Figure 11. The same elements (as shown in figure 10) dismantled and re-assembled into an orthogonal scheme with four-element junctions.

Figure 12. The assembly of a temporary structure composed of twenty-five reciprocal “short beams”
interconnected according to the Fullerian geodetic scheme (a pentagon formed of five hexagons).

The “prototype” was left in this composition as an example of experimental building, available for further structural verifications to be carried out.

The photographs shown in figures 10, 11, and 12 show some of the possible “assembly exercises” that were performed with the prototypal beams; being created, assembled and subsequently dismantled according to various schemes and geometries. The composition game, offered by such elements, is without limits and dependent only upon the circumstances of the support points and upon the geometry of the context.

7. THE COMPILATION OF AN EXEMPLARY PROJECT

To confirm the real applications of the technologies studied in the field, a project was elaborated and submitted to the competition announced in 2008 by the Italian Foreign Office for the construction of the Italian Pavilion for the 2010 World Expo in Shanghai. The project demonstrates the actual potential of a similar technology that was devised for the competition, which required for environmental sustainability reasons that the structure could be dismantled at the end of its temporary use and potentially reused in a different form and for a different function. The idea of the proposed project deliberately makes reference to the design of Leonardo in his Atlantic Code (fig 4) that he himself describes as “chains of bonds of wooden beams for the construction of a provisional building”.

Figure 13. The Leonardo model (cf. fig. 4) put together by students at the University of Bergamo.

The citation from Leonardo’s sheet referred to a matrix similar to the present project; thus, even in the evident shift to the more sophisticated technologies of today, it underlines the strong link of continuity that these structures convey with architectural traditions, Chinese construction, the history of engineering, modern Western architecture, and above all, with the Italian Renaissance and the marvellous construction inventions of Brunelleschi and Leonardo. The occasion presented by the competition, became an opportunity to study and exercise the application of our research and has permitted the drawing up of the project for a roof structure in square plan measuring 60 linear metres per side, able to hold a free space where the needs of the exposition could be arranged. The fact that the structure can be dismantled at the end of the exhibition in order to be reused in potentially different forms and dimensions, was possible because the entire structure consists exclusively of dry joints in the form of simple resting supports, i.e. without the use of bolts, welding, resins etc.. It instead takes advantage of the principle of structural reciprocity, the characteristic possessed by the composing elements. For this reason, the structure, at the end of its use at the exhibition, could be recomposed into other forms and dimensions using distinct support points. The individual beams once dismantled will be able to be recomposed into alternative geometries to the initial orthogonal scheme that would be employed: building in this way, allows the construction of triangular, pentagonal or free forms with more complex junction shapes.

The proposed project employs orthogonal junctions formed by four beams, and is demonstrative of just one of the solutions “possible”; the choice of the geometry chosen essentially derives from the wish to emphasize the reference to Leonardo’s scheme, but the same short beams could give rise to other buildings of alternative uses and contexts. To emphasize the pattern of the interweaving structure of reciprocal beams, the framework's waterproof cover, made in sheets of polyurethane, were designed to be attached on the underside of the structure (fig 14); thus leaving the interwoven structure of beams exposed to the outside and, at the same time, using the sheets as shading elements for the pavilion’s air conditioning and lessening the level of sun exposure to the inside of the structure. Technological innovation, architectural and structural engineering research and proposals of environmental sustainability are thus three aspects that find a happy fusion in this project: a fusion that is founded upon the character of the “reciprocal structure” itself and that seems to derive inspiration directly from organic and natural principles. Indeed, we would like to define the “sense” of reciprocity in structures as a sort of “natural architecture”, in that their forms seem to generate themselves from a process of self-reproduction and of spontaneous growth of similar and repetitive elements. This deep connection with nature is what has drawn the “reciprocal
structure” to the most innovative structural engineers of the last century: from R. Buckminster Fuller to E. Torroja, from P.L. Nervi to S. Musmeci. The peculiarities of such structures would be truly difficult to comprehend if one does not recognise the inspiration that arises from the attentive and detailed study of structures and materials and the observation of growth and organisational processes in cells within the organic world.

The principles, upon which the equilibrium of the reciprocal structure is founded, seem, in fact, to encounter very different subjects, from geometry to biology, in a suggestion of form from which descends the same innovative value of the system and its imaginative role.

If it were left to the structural engineers to take constructive imagination back into the heart of education and design, the architects and designers would find further confirmation of just how much the beauty of architecture is linked to the “unchanging”, to the materials used and to the internal life of architecture itself; whereas, innovations of shape and form in architecture are guilty, only too often, of just making bad reading.

8. CONCLUSIONS

One can conclude that the results of the studies, conducted and coordinated within the Faculty of Engineering at the University of Bergamo, do not only demonstrate the possibilities offered by this field of architectural and structural research and building production – that must continue to be investigated and technologically improved – but it has also allowed the emergence of beautiful new forms and compositions that these systems can offer. The junctions and the interweaving of the structural elements, that interact using simple “dry” joints, certainly emphasize the evocative image and timelessness that these structures project; so much so, in fact, that we are prompted to share the considerations of the studious Englishwoman Olga Popovic Larsen, who, to conclude her recent publication dedicated to reciprocal structures, asked whether it is not now the case – speaking about “Reciprocal Frames” – that the term “structure” should not be used anymore, but instead to more appropriately substitute it “tout court” with “architecture”. And so it is like this, that the challenge, thrown to us from Leonardo and the antiques, can and must be received by the architects and engineers of today, and it is in this sense that we can read into some of the more interesting proposals from contemporary architecture, from Shigeru Bahn to Cecil Balmond, Calatrava and the recent proposal of Frank O. Ghery for the Serpentine pavilion in London – to perceive how, as one says, they draw inspiration and found their “strength of image” into the large and changing sensation that this structural conception contains within itself.

Finally, given the relative “novelty” of the argument, I would also like to emphasize the significance of the bibliography, and give credit to those who advance the studies of these structures despite the lack of attention from the rest of the scientific community.
Figure 17. Frank O. Ghery; the pavilion of the Serpentine Gallery, London, Hyde Park, 2008.

Figure 18. Cecil Balmond; a model of a reciprocal structure, the Cecil Balmond exhibition, The Louisiana Museum of Modern Art, Copenhagen, July-Aug. 2007.

Figure 19. Santiago Calatrava; the Turning Torso skyscraper (background) and the reconstruction of a Scandinavian aboriginal construction (foreground) based on the static principle of the “spatial reciprocal structure”, Malmö - Sweden, 2005.

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Photography: all photographs were taken and belong to the author.
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